

ASSET HEDGING VIA DIGITAL ASSET INDICES²

In the contemporary financial landscape, numerous conventional market indices exhibit a pronounced correlation with the S&P 500, the universally acknowledged benchmark. Although contemporary non-institutional investors can access a wide array of indices, their capabilities to effectively hedge these investments remain constrained. This paper proposes expanding the existing options by incorporating non-standard asset classes, focusing on digital assets backed by blockchain technology. The key findings indicate that certain digital asset indices, despite their inherent volatility, can serve as viable hedging tools, presenting a lower correlation with the S&P 500 compared to traditional market indices. The study reveals that digital asset indices, particularly those optimized for liquidity and risk-adjusted returns, significantly enhance portfolio diversification and exhibit hedging efficiencies comparable to, or in some cases, superior to, conventional hedging instruments such as gold.

Keywords: portfolio hedging; cryptocurrency; market risk

JEL: G00; G10; G11

1. Introduction

Since 2019, a series of significant global events has continuously reshaped the economic landscape. The U.S. public debt increased by an estimated 25% due to government actions in response to a global pandemic. A considerable portion of Quantitative Easing funds was issued directly to citizens, spurring an uptick in equity investments, heightened market volatility, and liquidity challenges (Iyer & Simkins, 2022). A financial contagion is further observed in the aftermath of the Russia-Ukraine conflict, with time-varying connectedness observed among equity markets and alternative assets (Yousfi, Farhani, & Bouzgarrou, 2024).

The cryptocurrency sector also faced its share of instability, evidenced by the downfall of a notable stablecoin (UST, also known as Terra), a leading hedge fund (Three Arrows Capital or 3AC), and a significant exchange (FTX) within the span of a year (2022). Governmental regulations from the United States, Canada, Great Britain, Australia, and certain EU member states, alongside banking restrictions, have stymied the widespread adoption of digital assets. Notwithstanding these obstacles, digital assets have consistently produced higher average

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returns than traditional market indices based on mean returns, albeit without accounting for the associated higher volatility and risk. This assertion is supported by a risk-adjusted performance analysis using Sharpe ratios. This study will delve deeper into the risk-return dynamics of digital assets relative to traditional indices.

Nevertheless, the risks associated with digital assets are more complex and extend beyond mere price volatility, with market liquidity as a predominant concern (Gerunov, 2022, pp. 81-114). To mitigate these risks, we have developed nine different indices based on digital assets, aiming initially to maximize returns and minimize risks (Section 3.1). Subsequently, we evaluate a subset of indices for their efficacy as hedging instruments against the S&P 500 (Section 3.2) and suggest investment strategies that cater to various risk profiles.

We aim to address current gaps in scientific literature, which focuses on hedging primarily via a single asset – Bitcoin – and fails to provide a wider overview of risk and opportunity by using a cryptocurrency-based index.

We focus on the impact of dynamic correlation (DCC) and measure and compare hedging effectiveness since the correlation between stocks and cryptocurrencies and the volatility of these assets are time-varying (Fakhfekh & Jeribi, 2020) (Klein, Thu, & Walther, 2018) (Tiwari, Raheem, & Kang, 2019).

2. Literature Review

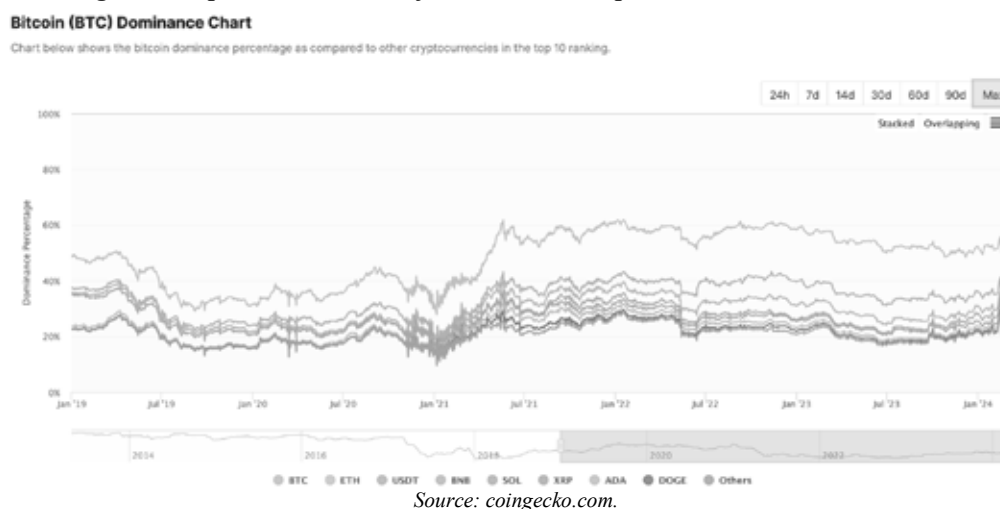
The concept of an index of cryptocurrency assets, serving as a global crypto index, remains a relatively underexplored area in academic literature, with most authors choosing to consider individual assets in their hedging analysis. This observation is principally attributed to the notable correlation observed among the majority of these assets with Bitcoin, in addition to the considerable market dominance exerted by Bitcoin itself (over 50% of the total cryptocurrency capitalization between 2021 and 2024), as depicted in Figure 1.

Limited liquidity of assets has been shown to hamper the diversification and hedging capabilities of cryptocurrencies (Moreno, Antoli, & Quintana, 2022) or even lead to anomalous behaviour of the digital asset (Dong, Lei, Liu, & Zhu, 2022). That's the rationale for our consideration of an index of cryptocurrencies as a hedging instrument, rather than a single – possibly illiquid – asset like Bitcoin.

Previous research has demonstrated that as few as five cryptocurrency assets could account for up to 0.9999 of the total cryptocurrency market as of 2019 (Shah, Chauhan, & Chaudhury, 2021). We observe this percentage going down in recent periods, both in terms of market capitalization and trading volume. However, the main utility we are pursuing through the usage of a cryptocurrency-based index is a larger overall market depth, especially from the perspective of asset liquidity. Spreading the hedging capability across assets aims to ensure the ability to enter, exit or calibrate positions in a more money-efficient manner. While of secondary concern, our analysis shows minimal variance in index performance when expanding beyond 12 to 15 assets. Moreover, adding assets with lower trading volume hampers liquidation options and heightens risk, rendering broader indices less effective for hedging. (Moreno, Antoli, & Quintana, 2022). Our first research goal thus becomes to

develop and explore the capabilities of cryptocurrency indices based on market capitalization, trading volume and risk (based on return variance). Once these alternatives are developed, we investigate their viability by considering index volatility, correlation to Bitcoin and Sharpe ratio to measure potential returns. Based on these factors, in the second part of the research, we select one index and compare its ability to hedge the S&P500 against other hedging instruments – market indices from different markets, gold and crude oil.

Figure 1. Top 10 Asset Share Of Total Market Capitalization Jan 2019-Jan 2024



To assess hedging, we first look at correlations between crypto assets and stock markets. Research by Zhenzhen Zhia et al. highlights a negligibly positive correlation between Bitcoin returns and those of the Chinese market during the period spanning Q3 2014 to Q2 2019. They further discern an investor predilection towards Bitcoin as a hedging mechanism “against extreme down movements in the Chinese economy and stock market index” (Jia, Tiwari, Zhou, Farooq, & Fareed, 2023), despite governmental prohibitions and restrictions in China’s mainland and territories. As part of our exploratory analysis, these findings are extended to an array of cryptocurrency indices that demonstrate an exceptionally low positive or negative Pearson correlation with the Shenzhen 225, the world's eighth largest market index, during the period from January 2019 to April 2023. While looking at correlations, we should also consider spillover effects from the stock market: Dimitrios Bakas et al. have identified the S&P 500 as a significant influencing factor. (Bakas, Magkonis, & Young Oh, 2022) While volatility does not typically exhibit a correlation with return correlations, our findings indicate that movements within the S&P 500 not only impact the volatility of blockchain-based assets but also influence their market direction. This relationship could potentially be attributed to the global policy of quantitative easing (QE), implemented as a response to the COVID-19 pandemic. Further scrutiny also reveals discernible periods of heightened and diminished correlation between traditional and digital markets. The variance of correlation over time, in turn, impacts the hedging efficiency and we explore and compare

that metric across asset classes. Looking further back at the relationships between stock and crypto, Anne Haubo Dyhrberg's analysis revealed that between 2010 and 2015, Bitcoin demonstrated no correlation with the Financial Times Stock Exchange (FTSE) index and exhibited only limited correlation with two significant foreign currencies – the British Pound and the United States Dollar (Dyhrberg, 2016). These attributes position the asset itself as an effective hedge against market indices, with a level of efficacy that is at least comparable to gold. That study, however, only includes Bitcoin, rather than the wider cryptocurrency universe, and considers a heterogeneous period of growth in traditional markets. In contrast, the period we observe contains significant periods of instability in both asset families. With that said, constructing a blockchain-based index and comparing its performance with the FTSE 100 yields similar outcomes even over a later period (2019-2024). Two noteworthy observations emerge from this comparison. First of all, the correlation appears not to significantly increase over time as digital assets gain wider acceptance among investors. Secondly, while the correlation with the FTSE is relatively minimal but positive, the S&P 500 exhibits a somewhat higher correlation with a blockchain index over the more recent period.

Looking at the overall correlation, especially at longer periods of time, obfuscates the short-term hedging capabilities and – based on our research – does not properly describe the hedging behaviour in situations of extreme short-term uncertainty. This, too, has been identified and widely researched in the past: In their empirical research, Debdatta Pal et al. undertook the task of calculating optimal hedge ratios between Bitcoin and a selection of other financial assets (Pal & Mitra, 2019). To achieve this, they employed conditional volatility estimates derived from a range of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, spanning the period from 2011 to 2018. Although their investigation primarily targeted the hedging of Bitcoin using a wide variety of assets, the methodological framework they adopted is transferrable and holds relevance to the research problem at hand. Supporting literature includes studies conducted by Kroner and Sultan, Arouri et al. and Basher and Sadorsky. Our research does not offer a new model or method, but merely compares the output of DCC- and ADCC-GARCH and compares the estimated hedging efficiency.

3. Data Sources

We use market data from two primary sources. The first is Yahoo Finance, accessed via Application Programming Interface (API) calls. This platform, in collaboration with CoinMarketCap, provides a diverse array of data relevant to this investigation. The second source is the website coindocex.com, from which historical market capitalization data was extracted by analyzing its quarterly historical records. Additional data sources (charts from messari.com, coingecko.com) are clearly denoted or mentioned in the footnotes.

The analysis primarily focused on the daily closing data of various indices from January 1, 2019, to January 1, 2024. Although blockchain transactions operate around the clock, market index data is conventionally available only for certain weekdays. To maintain consistency and comparability, both data sets were normalized to conform to the standard trading

calendar. It's worth noting that price variations in blockchain assets are treated as equivalent to off-hours trading in traditional exchanges. Furthermore, quarterly data related to the market capitalization of blockchain-based assets were compiled. This approach mirrors the typical rebalancing schedule seen in traditional market indices, thereby ensuring a methodologically congruent analysis across both digital and traditional asset classes.

4. Methodology

Initially, a series of blockchain indices is constructed, and the logic underpinning each selection or combination is thoroughly discussed. At its core, all choices are derived from practical investment options, and this research endeavours to transpose these investor alternatives within the realm of digital assets. Given that some of the alternatives have a very tight correlation with the leading asset or offer little additional utility in terms of liquidity or risk, we consider them not to hold practical value for the informed investor. Subsequently, the more meaningful alternatives are subjected to rigorous testing to ascertain their hedging potential. We explore the optimal hedging ratios and resulting portfolios based on GARCH first; due to the length of the period and the systemic shock observed in Q1 2021, we seek a more dynamic correlation between the asset and the hedge, to further assess hedging capabilities outside of the very long-term. For that, we employ DCC-GARCH – a model that extends the GARCH framework by allowing the conditional correlations between asset returns to vary over time. This dynamic correlation is crucial for hedging purposes as it enables the construction of a portfolio that can adapt to changing market conditions, thereby providing more effective risk mitigation. Additionally, we examine if the dynamic correlation is symmetric or not to deduce whether market conditions affect assets differently. We use ADCC-GARCH for that goal, but conclude that it does not produce qualitatively different outputs than DCC-GARCH and opt against using it to lower model complexity and prevent overfitting. Dynamic correlations are estimated at the quarterly level, to fit the cadence of index rebalancing. This approach aims to balance index utility with transactional cost and mirrors traditional stock exchange practices.

Finally, we engage in a comprehensive discussion of the obtained results.

4.1. Index size

To determine the optimal number of constituents for our indices, the following evaluative criteria are employed (Chan, 2021) (O'Donnell, 2022):

Diversification: The incorporation of a broader range of assets in an index can mitigate risk via diversification. However, the benefits of diversification may plateau once a certain asset threshold is surpassed. In the realm of cryptocurrencies, diversification is a specific topic: most leading assets exhibit a high level of return correlation and covariance (Figure 2), while the top two assets represent over 70% of the market capitalization (Figure 1).

Liquidity: It is essential that the assets incorporated in the index possess adequate liquidity. This requirement can pose a challenge when a larger number of assets are included,

particularly recently released tokens, which carry additional risks (e.g. “pump-and-dump” fraud). We use the quarterly mean trading volume as a liquidity indicator. Liquidity is, indeed, one of the leading reasons to build and test an index rather than an individual asset.

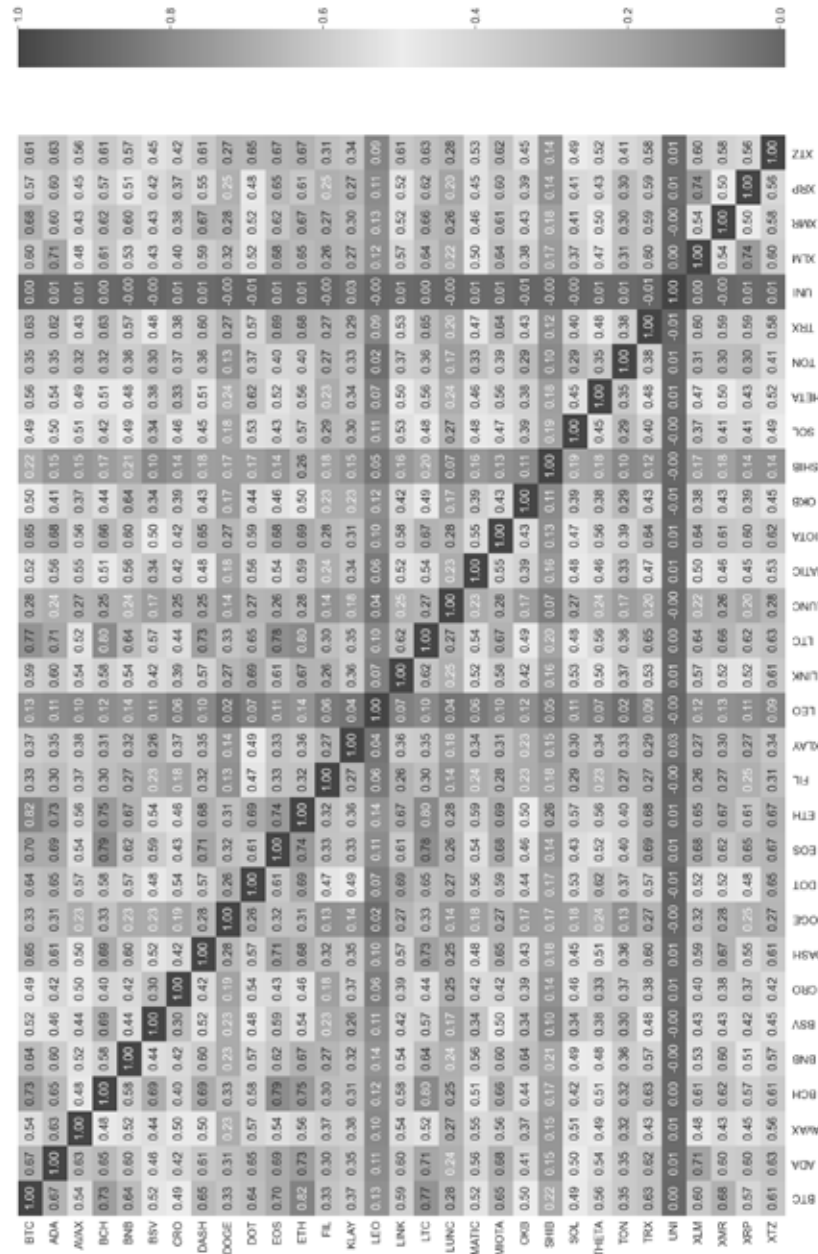
Rebalancing and Transaction Costs: The inclusion of more assets in an index may precipitate elevated rebalancing and transaction costs. Each rebalancing episode may necessitate the purchase or sale of assets, thereby incurring transaction costs. Tightly connected is another criterion: **Complexity and Manageability** of an index. An increase in the number of assets can render it difficult to manage, especially around entering, exiting and rebalancing operations. It is also important to note that different assets entail different transactional costs, making calculations more complex; this inconvenience could be mitigated by trading perpetual futures instead. The complexity of factoring in varying transaction costs on blockchains is out of the scope of the current research.

Tracking Error: The quantity of assets can also impact the tracking error, defined as the discrepancy between the performance of the index and a portfolio attempting to emulate it. While a larger number of assets can potentially diminish tracking error, it concurrently escalates the complexity and cost of replication. This criterion is of little importance for the goals of this research and is, therefore, ignored.

From our analysis of 31 distinct assets, we have established that just 10 thereof exhibit a correlation with the leading asset that falls below 0.5. This observation substantiates our anticipation that the resultant indices would likely maintain a positive correlation with Bitcoin. This is not a new finding in scientific literature, we are looking to build an index with two distinct goals: follow the larger cryptocurrency market (somewhat diverging from the Bitcoins movement), and ensure sufficient liquidity (cross-asset) is available to enable reactions to unfavourable market conditions. Therefore, the high correlation with the leading asset should not be perceived as a disadvantage.

This expectation also aligns with the research conducted by Agam Shah et al., who constructed a comprehensive market index solely incorporating five assets. Our study reveals no significant discrepancy in correlation when employing equal weights for five or more assets; however, the quantity of components included does influence overall profitability, Sharpe ratios, and risk (as gauged by standard deviation). Intriguingly, both risk and profitability appear to stabilize after incorporating 10 to 15 assets, contingent on the specific period under scrutiny and the weighting methodology implemented. Furthermore, assets with smaller market caps typically have lower liquidity, leading to higher price volatility, which limits their suitability for inclusion in a market index. Additionally, the market capitalization disparity between assets beyond the top 10 can be vast, sometimes spanning several orders of magnitude. Trading fees are also a consideration for investors constructing detailed indices. To circumvent the potential pitfalls of excessive volatility, potential liquidity shortages, and technological risks associated with blockchains (Chen, Chang, & Yang, 2023), indices are composed of twelve constituents at any point in time.

Figure 2. Pearson Correlation Matrix Of Assets Participating In Indices Within The Research; correlation is calculated on the returns



4.2. Asset exclusions

A deliberate choice to exclude "stablecoins" from index participation is made, as these assets essentially replicate dollar value and, in theory, should not confer any advantage to their holders (Li, et al., 2024). While stablecoins do represent US dollar liquidity and can be employed for market making – thus generating income based on usage – they do not directly yield dividends when held at an exchange. Furthermore, empirical evidence underscores their significant risks, primarily due to regulatory oversight deficiencies. As such, a stablecoin asset would merely introduce risk to a cryptocurrency index and is thus not deemed a desirable constituent. This claim is exasperated by the expected regulatory action against cryptocurrencies pegged to the US dollar.

Additional exclusions have been considered based on criteria analogous to those implemented for S&P 500 selection. This includes a requirement for a public float of at least 10% of the asset, as well as liquidity requirements. Specifically, the ratio of annual dollar value traded – calculated as the aggregate shares traded over the preceding 12 months, multiplied by the float-adjusted price as of the index rebalancing reference date – to the float-adjusted market capitalization should be equal to or greater than 1.00. These prerequisites were enforced wherever ample trade data was accessible, and it's noteworthy that their application did not yield significant impacts on the dataset.

4.3. Weights of participating assets

Traditionally, the construction of market indices is conducted to mitigate or eliminate inherent risks such as individual stock risks, liquidity risks, and price movement risks. Risk management is typically addressed most effectively through the allocation of diverse weights in an attempt to counterbalance the underlying risk, hence, making it a crucial variable in the process of index construction.

Three primary weighting strategies are taken into consideration in this study: a strategy based on total market capitalization, another predicated on risk balancing (as reflected by the standard deviation of assets in the preceding quarter), and a final strategy hinged on market liquidity. Risk balancing has been selected to address well-known digital asset volatility (Bakas, Magkonis, & Young Oh, 2022) (Baur, Lai, & Hossain, 2022), whereas market liquidity has been identified as a key crypto currency risk and even added to risk management methodologies (Gerunov, 2022, pp. 81-111) Strategies reliant on price weighting are deliberately excluded from this study, as this approach exhibits markedly distinct dynamics in contrast with traditional markets. Given the impossibility of stock share splits or aggregations with blockchain technologies, and the presence of extreme price differentials, the practicality of price weighting is significantly diminished.

The total market capitalization strategy is a reference to the way many traditional market indices like the S&P500 are created and rebalanced over time³. While these mechanics make sense in traditional markets, especially very liquid ones (S&P 500 total capitalization is \$43T

³ <https://www.spglobal.com/spdji/en/idsenhancedfactsheet/file.pdf?hostIdentifier=48190c8c-42c4-46af-8d1a-0cd5db894797&indexId=340>.

and an average daily trading volume of just over \$4B)⁴, we want to account for two of the hallmarks of cryptocurrencies – liquidity and volatility – that may have significant effects on our model (Gerunov, 2022, pp. 81-111) (Dong , Lei, Liu, & Zhu, 2022). Another important aspect is the dominance of the leading two assets (Bitcoin and Ethereum) on the offered instruments. High correlation and high market cap dominance would mean that the effect of the bottom 10 assets is trivial. To counterweight that notion, we also offer caps on some of the indices and explore the resulting behaviour both from the perspective of correlation with Bitcoin, but also from the risk/return standpoint. Caps appear arbitrary, but their variation is explained only by the ambition to meaningfully include as many assets as possible.

Additionally, we introduce both static and dynamic investment universes as frameworks for constructing and evaluating digital asset indices. A static investment universe entails selecting a fixed set of assets at the outset of the observation period, without changing them over time. This approach assumes that the initially chosen assets will continue to represent a viable investment strategy throughout the study period. It is particularly appealing for conservative investors seeking stability and predictability in their investment choices, as it avoids the need for frequent adjustments and the associated transaction costs. On the other hand, practice shows that many assets once deemed valuable have lost much of their market value and trading volume. Conversely, a dynamic investment universe involves periodically reevaluating and adjusting the composition of the index to include assets that currently meet the selection criteria based on market capitalization, liquidity, and risk parameters. This method allows the index to adapt to changing market conditions by incorporating emerging assets that may offer higher returns or better diversification benefits and excluding those that no longer fit the desired profile. Although a dynamic universe can potentially enhance certain index capabilities, it requires more active management and may incur higher transaction costs due to the frequent rebalancing. By offering both static and dynamic investment universes, we aim to cater to different investor preferences and risk tolerances. The static approach suits investors who prefer a more hands-off, buy-and-hold strategy, while the dynamic approach appeals to those willing to engage in more active management to potentially capitalize on market trends and shifts in asset performance.

A word of caution: static indices lose the majority of their constituents within three years due to market dynamics; while by the end of the observed period, some assets reenter the Top 12 of the respective category, limiting the number of constituents goes against the goal of higher liquidity, and is therefore discouraged by the authors – it only serves to compare performance and risk.

Here is the final proposed weight selection strategy of the nine indices:

⁴ As of Jan 1, 2024; Source: Yahoo!Finance data;

Table 1. Indices used throughout the research

Identifier	distribution	allocation basis	total # of assets ⁵	rebalancing
MCAP_WEIGHTS_NAIVE	naïve / equal ⁶	market capitalization	12	none
MCAP_STATIC_ASSETS_DYNAMIC_WEIGHTS	proportional	market capitalization	12	quarterly
MCAP_STATIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	proportional, capped at 27.5%	market capitalization	12	quarterly
MCAP_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS	proportional	market capitalization	31	quarterly
MCAP_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	proportional, capped at 12.5% ⁷	market capitalization	31	quarterly
LIQ_STATIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	proportional, capped at 16.7%	trading volume	12	quarterly
LIQ_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	proportional, capped at 20%	trading volume	31	quarterly
RISK_STATIC_ASSETS_DYNAMIC_WEIGHTS	proportional	risk	12	quarterly
RISK_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS	proportional	risk	31	quarterly

Table 2. Descriptive statistics of the observed indices, including risk/return, market beta

Index	#	risk	skew	kurtosis	JB Test	Returns	Sharpe Ratio	Market Beta
S&P 500	1082	0.014	-0.67	13.218	7878.481	0.884	0.772	1.001
NASDAQ-100 Technology Sector	1082	0.021	-0.215	4.155	777.327	1.585	0.84	1.284
Dow Jones	1082	0.014	-0.739	18.877	16005.06	0.608	0.611	0.951
NASDAQ Composite	1082	0.016	-0.463	6.782	2089.432	1.228	0.848	1.097
FTSE 100	1082	0.012	-0.786	13.343	8056.981	0.131	0.247	0.508
Russel 2000	1082	0.018	-0.676	8.403	3231.821	0.468	0.455	1.129
DAX	1082	0.014	-0.284	12.967	7518.168	0.556	0.575	0.629
Shenzhen index	1082	0.015	-0.239	2.64	320.142	0.307	0.386	0.21
Nikkei 225	1082	0.013	0.208	3.673	608.761	0.711	0.712	0.231
Crude Oil	1082	0.108	-22.343	612.204	>16M	0.494	-0.344	1.06
Gold	1082	0.011	-0.098	4.269	813.618	0.608	0.742	0.051
MCAP_WEIGHTS_NAIVE	1082	0.047	-0.208	7.493	2511.743	9.349	1.111	0.982
MCAP_STATIC_ASSETS_DYNAMIC_WEIGHTS	1082	0.047	-0.121	7.36	2418.436	11.068	1.155	0.972
MCAP_STATIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	1082	0.058	4.029	72.065	234858.4	12.279	1.076	0.966
MCAP_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS	1082	0.048	-0.091	7.393	2439.3	10.415	1.134	0.973
MCAP_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	1082	0.047	-0.188	7.504	2517.9	9.609	1.119	0.98
LIQ_STATIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	1082	0.047	-0.193	7.476	2499.714	9.720	1.123	0.98

⁵ A list of assets and weights is available upon request.

⁶ A static weight allocation is maintained throughout the entire period, provided that assets remain available for trade. All assets in the top 12 since 2019 still tradeable at present. The index price is subsequently normalized to ensure continuity.

⁷ Weights capped at 12.5% to prevent the dominance. Throughout the majority of the observed period, Bitcoin (BTC) and Ether (ETH) have a cumulative weight of 25%, which inhibits them from overpowering the index.

It is crucial to note that in order to achieve a continuous index price, quarterly rebalancing weights post-rebalancing — must also be calculated to compensate for varying asset prices. Jarque-Bera test indicates that the indices' returns are not following normal distribution, which is the expected behaviour.

4.4. Assessing hedging capabilities

Since its introduction, GARCH is widely adopted by both practitioners and scholars alike to investigate (among many other applications) asset correlation, covariation and overall hedging ability over time. To that effect, a multitude of extensions to the original model were created (Bauwens, Laurent, & Rombo, 2006). GARCH and its various iterations provide a comprehensive framework for evaluating an index's potential as a hedge against the S&P 500 (Engle R. , 2001). This approach allows for a thorough examination of the asset's volatility dynamics, its correlation with the S&P 500, its risk-return trade-off, and its performance stability under different conditions.

The first step in the process involves examining the conditional variances (and standard deviations) generated by the GARCH (Engle R. F., 1982). These conditional variances, indicative of an index's future volatility, are instrumental in discerning the index's hedging potential under diverse market conditions. A low and stable conditional variance might be indicative of the index's reliability as a hedge – in our case, hedging against the S&P 500. To further investigate the hedging potential, a multivariate GARCH model is estimated for the asset and the S&P 500 to evaluate the correlation of returns (Bollerslev, Engle, & Wooldridge, 1988). The mathematical representation of the theoretical construct follows:

In GARCH, the conditional variance $h(t)$ is defined as:

$$h(t) = \omega + \alpha \varepsilon_{i-t}^2 + \beta h_{i-t} \quad (1)$$

- ω is a constant term,
- α is the coefficient for the lagged squared residual term ($\varepsilon^2(t-1)$) capturing the impact of shocks,
- β is the coefficient for the lagged variance term ($h(t-1)$) capturing the persistence of variance,
- $\varepsilon(t-1)$ is the residual from the mean equation at time $(t-1)$,
- $h(t-1)$ is the conditional variance at time $(t-1)$

Through Eq. 1 we can easily define the conditional standard deviation $\sigma(t)$:

$$\sigma(t) = \sqrt{h(t)} \quad (2)$$

Once we have computed the standardized residuals

$$z = \varepsilon/\sigma \quad (3)$$

we can construct the second step of the process – the Dynamic Conditional Correlation (DCC) model:

$$H_t = D_t * R_t * D_t \quad (3)$$

where H_t is a $n \times n$ conditional covariance matrix (see Eq. 1), R_t is the conditional correlation matrix, and D_t is a diagonal matrix with time-varying standard deviations on the diagonal:

$$\begin{aligned} D_t &= \text{diag}(h_{1,t}^{\frac{1}{2}}, \dots, h_{n,t}^{\frac{1}{2}}) \\ R_t &= \text{diag}\left(q_{1,t}^{-\frac{1}{2}}, \dots, q_{n,t}^{-\frac{1}{2}}\right) * Q_t * \text{diag}(q_{1,t}^{-\frac{1}{2}}, \dots, q_{n,t}^{-\frac{1}{2}}) \end{aligned} \quad (4)$$

Finally, Q_t is a symmetric positive definite matrix representing the symmetric dynamic process, and is defined as follows:

$$Q_t = (1 - \theta_1 - \theta_2) * \bar{Q} + \theta_1 z_{t-1} z'_{t-1} + \theta_2 Q_{t-1} \quad (5)$$

$\bar{Q}$ is the $n \times n$ unconditional correlation matrix of the standardized residuals (see Eq. 3). The parameters θ_1 and θ_2 are non-negative. These parameters are associated with the exponential smoothing process that is used to construct the dynamic conditional correlations. The DCC model means reverting as long as $\theta_1 + \theta_2 < 1$; $\theta_1, \theta_2 \geq 0$. ADCC-GARCH, on the other hand, assumes that Q is asymmetric, and defines its dynamics as follows:

$$Q_t = (\bar{Q} - A' \bar{Q} A - B' \bar{Q} B - G' \bar{Q}^- G) + A' z_{t-1} z'_{t-1} A + B' Q_{t-1} B + G' z_t^- z_t'^- G \quad (6)$$

Where A , B and G are $n \times n$ parameter matrices, z_t^- are zero-threshold standardized errors, equal to z_t when less than zero and zero otherwise. $\bar{Q}$ and $\bar{Q}^-$ are the unconditional matrices of z_t and z_t^- . Finally, we're able to compare the optimal hedge ratios H^* of the three GARCH models:

$$H^* = \rho * \frac{\sigma_A}{\sigma_B} = \frac{\text{Cov}(R_A, R_B)}{\text{Var}(R_B)} \quad (7)$$

Where ρ is the conditional correlation between the returns on our asset (in this case, the S&P 500 index) and the hedging instrument (in this case, the blockchain index), σ_A is the standard deviation of returns on our asset and σ_B represents the standard deviation of our hedging instrument. This calculation is equivalent to the calculation of market beta, as R_B in this case, equals S&P 500 returns. Because of our use of GARCH, however, we can account for the time-varying nature of the variances and covariances of the assets. For a pair of assets, the optimal hedge ratio can be calculated as:

$$H^* = \frac{\text{Cov}_t(R_{A,t+1}, R_{B,t+1})}{\text{Var}_t(R_{B,t+1})} \quad (8)$$

This ratio relies on conditional second-order moments of asset and hedge returns that have emerged, where Var_t and Cov_t represent conditional variance and covariance at time t , and $R_{A,t+1}$, $R_{B,t+1}$ represent S&P 500 and blockchain index returns from time t to $t+1$, respectively. (Lai, 2019) The variance is simply the square of the conditional standard deviation:

$$\text{Var}_t(R_B) = \sigma_{R_{B,t}}^2 \quad (9)$$

A 90-day step is employed in order to converge with the rebalancing windows and limit rebalancing costs. While arguments may be made that a shorter step window may capture the asset covariance significantly better (refer to Figure 4), asset rebalancing cost might offset these gains. Hedging through perpetual blockchain futures may remedy the higher cost, but such a strategy is outside the scope of this research. We conduct a Portfolio Variance calculation for each 2-asset portfolio (S&P 500 vs. hedge) to understand the risk portfolio over time under optimal conditions. To do so, we'll apply the formula for the variance of a two-asset portfolio:

$$\begin{aligned} \text{Var}_p^2 &= \text{Var}_t(R_A)^2 + (H^*)^2 * \text{Var}_t(R_B)^2 + 2 * H^* * \text{Cov}_t(R_A, R_B) \\ \text{Var}_p &= \sqrt{\text{Var}_t(R_A)^2 + (H^*)^2 * \text{Var}_t(R_B)^2 + 2 * H^* * \text{Cov}_t(R_A, R_B)} \end{aligned} \quad (10)$$

The portfolio variance measures the dispersion of portfolio returns around the mean, serving as a proxy for risk. A higher variance indicates a higher level of risk, meaning the returns of the portfolio are more spread out over a wider range of outcomes. Conversely, a lower variance suggests that the portfolio's returns are more tightly clustered around the mean, indicating lower risk. By examining how portfolio variance changes over time, we identify periods of increased or decreased risk. We've identified the global COVID pandemic as a fundamentally disruptive event causing spikes in all observed portfolio variances, be it to a different magnitude.

In assessing the effectiveness of an asset as a hedge, the robustness of its hedging capabilities should be evaluated under various market conditions and over different time periods. This critical examination involves applying the GARCH model to historical asset data spanning several years. By applying this model, we investigate the volatility dynamics of the asset and its correlation with potential hedged assets, offering insights into its stability and reliability as a hedging instrument. This process describes how the asset responds to fluctuations in market conditions and whether its hedging properties remain consistent over time. To ensure a comprehensive evaluation, we have selected a time frame that encompasses a variety of global and localized stress events, impacting both traditional financial markets and the blockchain domain. This period includes significant economic downturns, geopolitical tensions, and regulatory changes affecting digital assets. Further including events with far-reaching effects in the blockchain domain is crucial for understanding the assets' behaviour

in response to industry-specific developments⁸. The impact of such events on the asset's volatility and correlation with other financial instruments can significantly influence the hedging effectiveness of an index. Finally, such events can lead to increased market volatility and act as good tests of the resilience of hedging strategies. By observing how the asset's hedging potential varies during these stress periods, we gauge its reliability across a variety of market scenarios. Through this detailed analysis, we aim to uncover patterns and relationships that may not be apparent in more stable market conditions, providing deeper insights into the asset's hedging utility.

In conclusion, testing the robustness of an asset as a hedge through the application of GARCH-based models over an extended period, inclusive of various market conditions, is essential for determining its efficacy and reliability as a hedging tool. This comprehensive approach allows us to understand performance nuances and offers valuable perspectives on its potential to serve as an effective hedge against market risks.

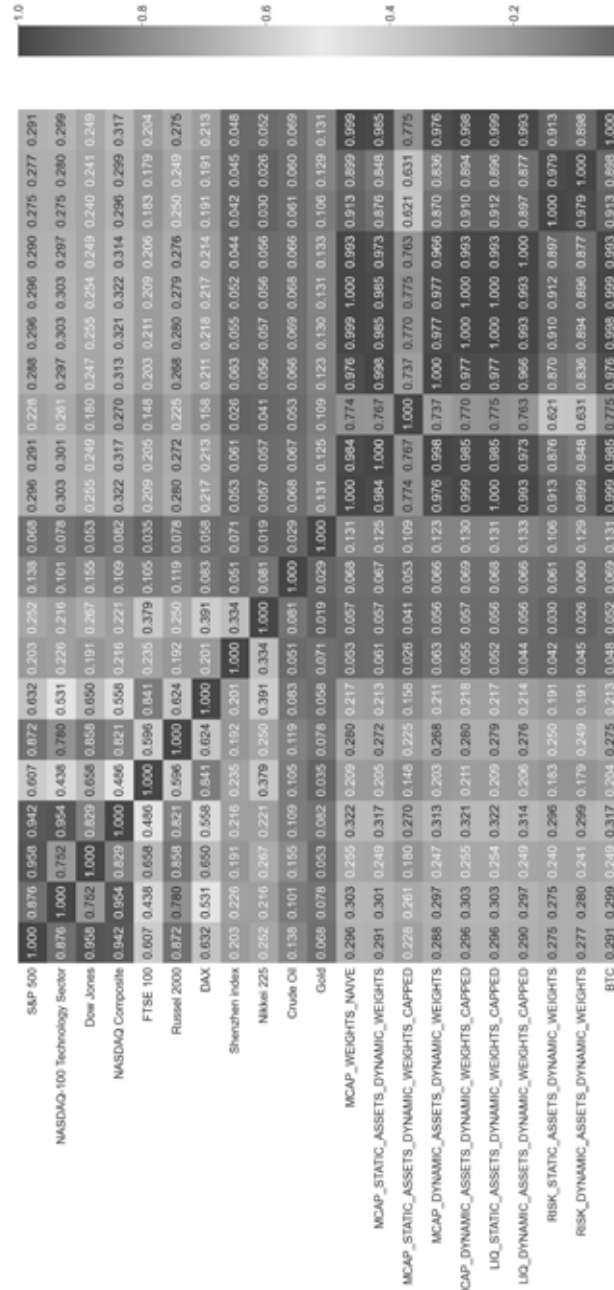
5. Findings

Once the index values as well as daily returns are computed for the duration of the period, the correlation between market indices and indices derived from blockchain assets can be observed:

Four distinct clusters are discernible (Figure 3). The first one is composed of market indices of Western economies. They consistently exhibit a relatively robust positive correlation (0.47-0.96). This high correlation renders them suboptimal candidates for hedging the returns of the S&P 500. Intriguingly, despite the expected interconnectedness of US indices, particularly in the wake of the extreme macroeconomic responses to the COVID-19 pandemic, the substantial correlation with the FTSE 100 and DAX is unexpected. Various interpretations could be posited, one of which might involve the global financial market's reliance on the US dollar. The second cluster contains market indices of Eastern economies. Here we observe a weak positive correlation with Western indices (0.14-0.36) and an insignificant correlation with blockchain assets. The dissociation of Asian indices might be explained in a variety of ways, including potentially more prudent central bank policies. Further examination reveals these indices exhibited the lowest returns over the observed period, coupled with the lowest Sharpe ratio. This makes them the least appealing investment from an investor's perspective, barring the hedging opportunity. The third cluster – precious metals and crude oil – appears to have the lowest average correlation with market indices both in Western and Eastern economies. Gold especially has been shown as an efficient and effective hedge of Western markets, even showing “safe heaven” properties – defined as a security that is uncorrelated with stocks and bonds in a market crash (Baur & Lucey, Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold, 2010) (Baur & McDermott, 2010)

⁸ Events include major hacks, technological advancements, or significant swings in market sentiment within the cryptocurrency space.

Figure 3. Correlation between daily returns of market indices and blockchain-based indices



With that knowledge in mind, these findings are not new and gold is only representative of the optimal hedging instrument. Lastly, market indices predicated on digital assets show a moderate positive correlation with Western market indices (0.2-0.3). Despite employing a wide range of diversification approaches, these indices display very high intercorrelations and a strong correlation with the leading asset – Bitcoin (BTC) (0.78-0.99), with one notable exception. Notably, the blockchain-based indices offer the highest returns in exchange for the greatest risk. However, they also exhibit the highest risk/return potential, as indicated by the Sharpe ratios (see Table 2). Given the consistency of these findings and the high correlation among the returns of blockchain-based assets, further exploratory endeavours, such as Monte Carlo simulations, are unlikely to yield qualitatively dissimilar outcomes.

When examining the skewness of returns (see Table 2), it is noticed that almost all indices under observation, with the exception of Nikkei 225, exhibit slight negative skewness. This indicates a greater likelihood of extreme positive returns compared to extreme negative returns. The relatively elevated kurtosis values suggest that the distribution of returns is acutely peaked and has fatter tails than would be expected in a normal distribution. This implies a heightened probability of extreme returns, both positive and negative, relative to a normal distribution. In financial parlance, this could indicate an increased risk associated with the investment, as extreme returns could lead to substantial gains or losses. Moreover, it could also hint at the exposure of the underlying assets to more pronounced and frequent price fluctuations, which could complicate the prediction of future returns. In summary, both skewness and kurtosis values are comparable across indices. Their behaviour generally aligns with expectations given the economic volatility experienced over the past five years.

When considering the balance between risk and return, traditional indices typically exhibit modest Sharpe ratios, generally fluctuating between 0.4 and 0.6. Over extended periods, the Russell 2000 typically resides on the lower end of this spectrum (0.2-0.4), while the NASDAQ Composite leans towards the higher end (0.5-0.7). Despite the prevailing consensus that investments in market indices constitute low-risk, low-return instruments, a quantification of this ratio reveals that for each invested dollar, the exposure to risk is precisely double the return opportunity. Observations made during the period spanning from 2019 to 2023 align with the boundaries of the aforementioned ranges. Contrary to that, indices predicated upon digital assets present a divergent risk-return dynamic. While the average risk associated with these assets can be up to three times greater, the potential returns tend to overcompensate, resulting in an overall Sharpe ratio that fluctuates between 0.88 and 1. This markedly higher value renders this opportunity considerably more attractive. However, given the high levels of volatility, these values may be susceptible to longer-term alterations.

An intriguing observation, necessitating additional exploration but exceeding the boundaries of the present study, relates to the shifting correlation dynamics between the S&P 500 and blockchain-based indices (Figure 4). An examination of the rolling correlation over a 90-day rolling window reveals a consistent upward trend, punctuated by an atypical surge during the first half of 2020. This surge aligns with the initial lockdowns and the macroeconomic shocks triggered by the COVID-19 pandemic. Moreover, it hints at an ongoing process of convergence between traditional markets and digital asset markets, potentially making blockchain-based assets less of an effective hedging opportunity. Possible explanations could

involve the overutilization of the asset as a hedge or investment spillovers from conventional markets.^{9 10}

Figure 4. Correlation dynamic between S&P500 and BTC; similar dynamics are observed with all derived indices



Based on the above findings, the decision is made to further refine the experimental scope by focusing on a smaller subset of crypto indices. This choice is driven by our earlier discussion on the high correlation between cryptocurrency-based indices. We choose indices with consistent performance, relative ease of construction, manageability, and stability in their risk-to-return profile. Additionally, we also consider Sharpe ratios as a factor of optimized risk-return ratio.

The objective of the remaining research is to evaluate the effectiveness of the chosen crypto indices as a potential hedge against the S&P 500. “Static” hedging means using a consistent hedge ratio over the whole period (see Figure 5), while the “Dynamic” approach uses 90-day windows to estimate the optimal hedging ratios. 1-period-ahead forecasting is also possible based on the selected models, but is left as a topic of future research.

While we observe a very low initial correlation between Gold and the S&P 500, GARCH is unable to produce a hedging ratio so that the correlation of the resulting portfolio is significantly different from the S&P 500. Optimally hedging via a static ratio demonstrates comparable results over the observed period with all crypto indices, as well as the core asset – Bitcoin. We observe the lowest optimal hedge ratio with Gold, Bitcoin and the crypto index trailing close by. The last asset group that we use for reference purposes – the Eastern industrial stock indices – exhibit both a higher optimal hedge ratio and higher temporal variation, making them an undesirable option for the rational investor. Our analysis, however, does not stop there – we also consider the correlation of the resulting portfolio with the initial asset – the S&P 500 market index. It is quite clear that by introducing such a small amount of the hedging asset, the correlation of the hedged portfolio with the main asset would be

⁹Due to the decentralized and pseudonymous characteristics of blockchains, distinguishing institutional investments may pose a significant challenge, thereby rendering further analysis unfeasible.

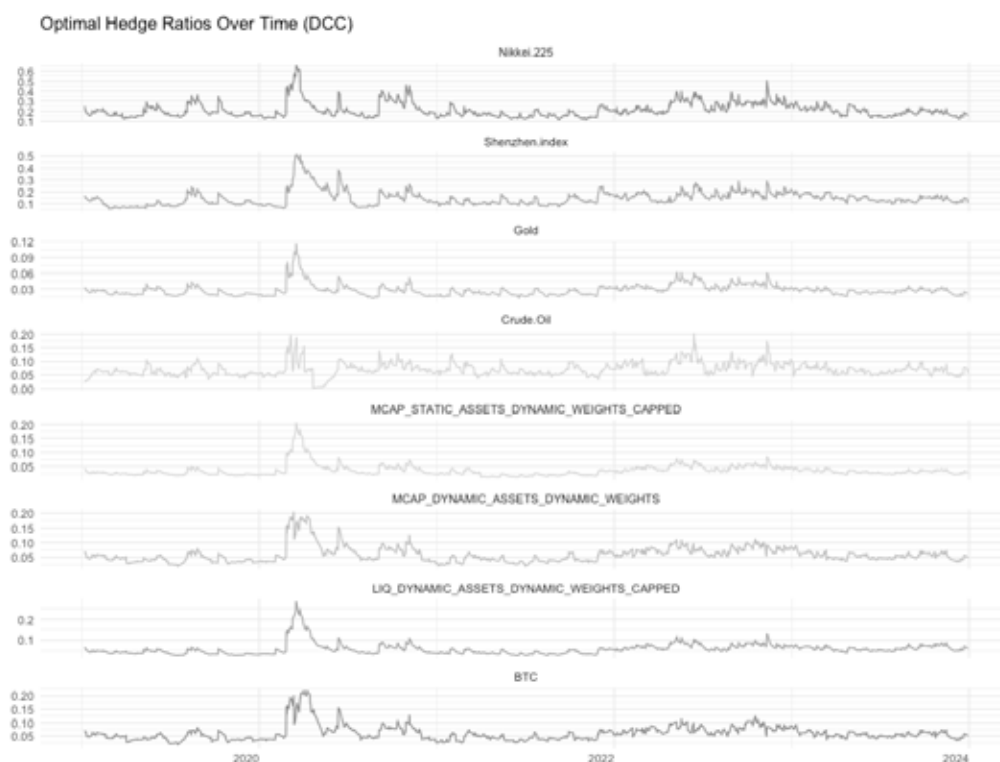
¹⁰ Bitcoin’s spot ETF has been approved in January 2024, further enabling the connection between traditional markets and digital assets; examining the spillover effects between the stock market and Bitcoin over time is a potential next research step.

insufficiently influenced. We, therefore, conclude that using a single optimal hedge ratio for the whole 5-year period of observation is of little practical utility; indeed, by developing a DCC-GARCH model, we are able to observe significant deviations from the mean of the optimal hedge ratio over time and across assets:

Table 3. Hedging efficiency and resulting returns based on different strategies with a static hedge ratio

Asset	Hedging Asset	Optimal Hedge Ratio	Original Correlation	Original Beta	Hedged Correlation	Hedged Beta	Hedging Efficiency (%)
S&P 500	NASDAQ-100 Technology Sector	0.602	0.876	1.283	0.471	0.227	76.734
S&P 500	Dow Jones	0.966	0.958	0.95	0.287	0.083	91.744
S&P 500	NASDAQ Composite	0.809	0.942	1.096	0.336	0.113	88.78
S&P 500	FTSE 100	0.726	0.607	0.508	0.795	0.631	36.876
S&P 500	Russel 2000	0.676	0.872	1.128	0.486	0.237	76.079
S&P 500	DAX	0.635	0.632	0.628	0.776	0.601	39.889
S&P 500	Shenzhen index	0.196	0.203	0.21	0.979	0.959	4.141
S&P 500	Nikkei 225	0.275	0.252	0.231	0.968	0.936	6.365
S&P 500	Crude Oil	0.018	0.138	1.060	0.99	0.981	1.910
S&P 500	Gold	0.091	0.068	0.051	0.998	0.995	0.463
S&P 500	MCAP_WEIGHTS_NAIVE	0.088	0.296	0.982	0.956	0.913	8.788
S&P 500	MCAP_STATIC_ASSETS_DYNAMIC_WEIGHTS	0.086	0.291	0.972	0.958	0.916	8.465
S&P 500	MCAP_STATIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	0.054	0.228	0.956	0.974	0.949	5.196
S&P 500	MCAP_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS	0.085	0.288	0.973	0.958	0.918	8.314
S&P 500	MCAP_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	0.088	0.296	0.98	0.956	0.914	8.755
S&P 500	LIQ_STATIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	0.088	0.296	0.98	0.957	0.914	8.762
S&P 500	LIQ_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS_CAPPED	0.085	0.29	0.971	0.958	0.917	8.396
S&P 500	RISK_STATIC_ASSETS_DYNAMIC_WEIGHTS	0.074	0.275	1.007	0.962	0.925	7.559
S&P 500	RISK_DYNAMIC_ASSETS_DYNAMIC_WEIGHTS	0.073	0.277	1.035	0.962	0.924	7.647
S&P 500	BTC	0.087	0.291	0.959	0.958	0.917	8.446

Figure 5. Optimal hedge ratios over time, based on DCC-GARCH analysis.



5.1. Considering correlation asymmetry

Based on the variance of the optimal hedge, one particular event stands out in the spring of 2020 – the COVID lockdown and the resulting economic shock. It is thus worth exploring if an asymmetrical dynamic conditional correlation exists: whether there are differences between the effects of positive and negative market movements on the correlations between assets. This feature is particularly relevant in financial markets, where negative shocks (e.g., market downturns like the COVID lockdown aftermath) often lead to a higher increase in correlations between assets than positive shocks do, a phenomenon known as "correlation breakdown" during crises. We therefore employ an ADCC-GARCH model and compare its optimal hedge ratios with the output from DCC-GARCH. Both models yield remarkably consistent ratios when calculating optimal hedge ratios. This consistency is observed not only between the S&P500 and our proposed index, but across the board, as per the rolling window method applied for forecasting conditional volatility (one-period-ahead) which is used to generate day-ahead hedge ratios, in line with Basher and Sadorsky's approach. Indeed, looking at the delta in values between the DCC- and ADCC- models, the largest difference across all assets equals $1.2e^{-6}$, which we do not consider significant. This observation may

lead to two plausible interpretations: first, there may be an absence of asymmetric effects. This suggests that positive and negative shocks do not have "leverage" effects and hence exert similar impacts on returns. Alternatively, it is also possible that both models adeptly capture the dynamic changes in correlations over time. Similar dynamics of the correlations produced by both models may indicate that the underlying correlation structure within the data is relatively stable and is effectively encapsulated by both models. Further investigation into the reasons why asymmetry appears to be minimal is suggested as a standalone research topic.

Figure 6a. Optimal hedge ratio calculation using DCC- and ADCC-GARCH

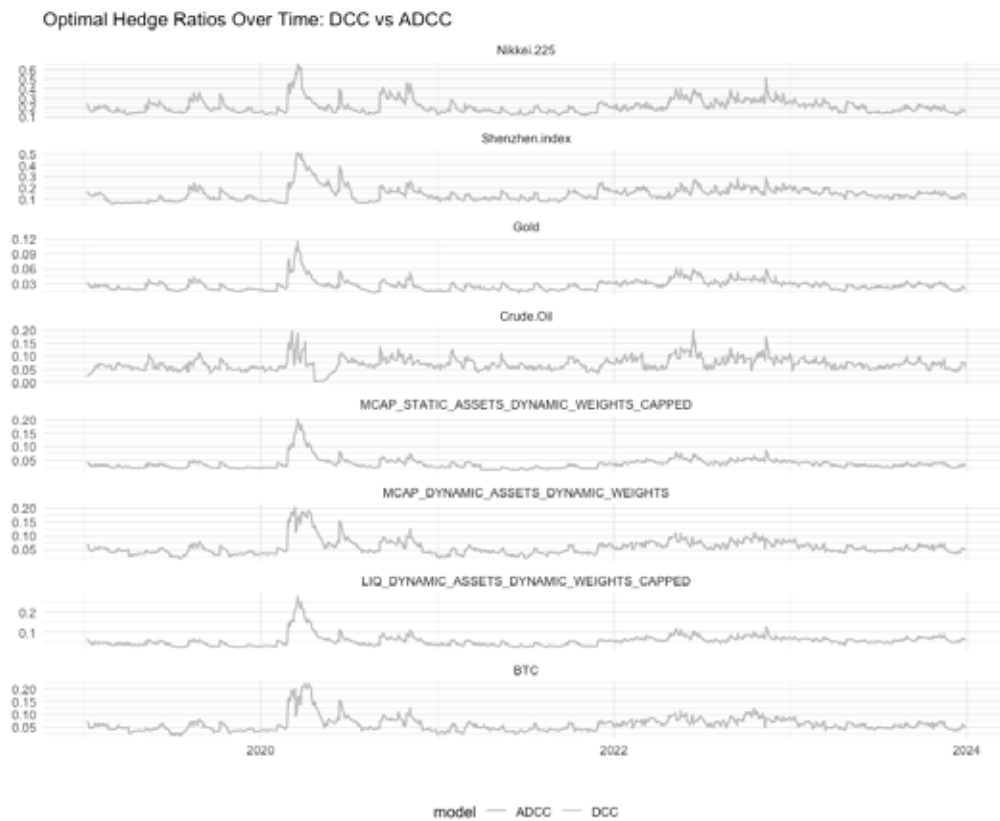
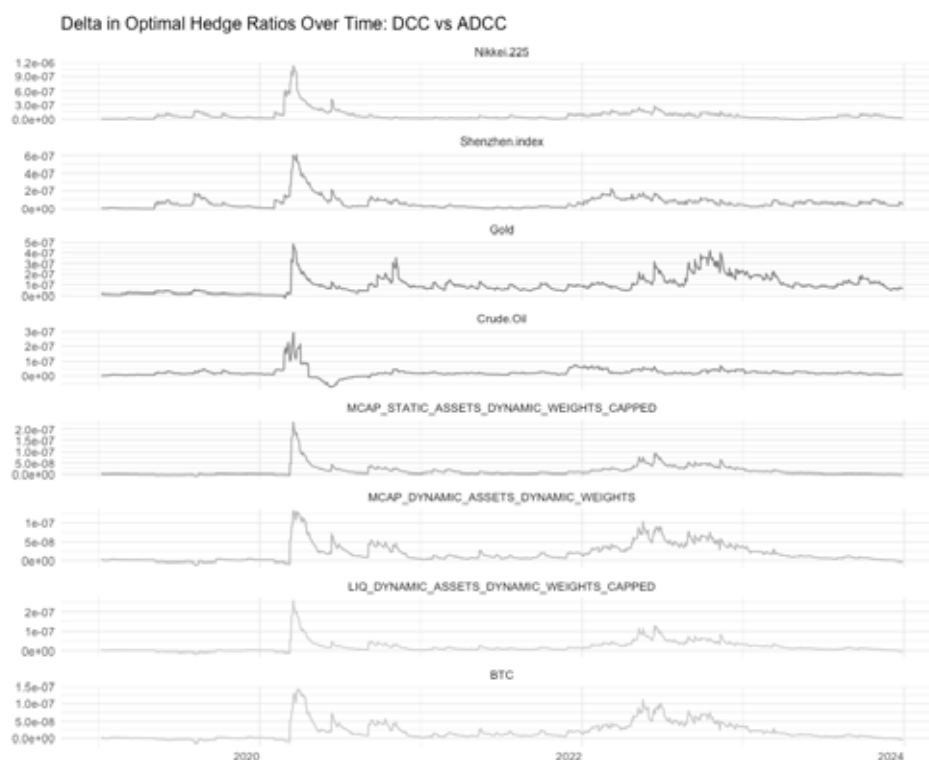


Figure 6b. Delta in optimal hedge ratio calculation using DCC- and ADCC-GARCH



Lastly, two practical considerations must be mentioned, that may sway the investor choice. First of all, changing hedge ratios can also imply changing transaction costs. A higher hedge ratio means you need to buy more of the hedging instrument, potentially increasing the cost of hedging. Another important point is to evaluate the effectiveness of the hedge (how well it reduces portfolio risk) against its cost is crucial. The optimal hedge ratio aims to balance these considerations to achieve cost-effective risk reduction. While these considerations hold practical importance, they vary across investors and are out of the scope of the current study. We opt to rebalance the hedge portfolio at the same cadence we rebalance the crypto index – once per quarter. Comparing portfolio variances over time, we observe the quarterly variance to reflect the optimal variance of the DCC-GARCH (Figure 7a and b). As we are averaging the effects, the variance at a quarterly level even appears lower than the peak point-in-time variance of the optimal fit; while this effect appears beneficial towards our goal, it is only true for longer-term hedging – investors are still vulnerable to short-term systemic shocks, as demonstrated in Q1 2021.

The portfolio variance measures the dispersion of portfolio returns around the mean, serving as a proxy for risk. A higher variance indicates a higher level of risk, meaning the returns of the portfolio are more spread out over a wider range of outcomes. Conversely, a lower variance suggests that the portfolio's returns are more tightly clustered around the mean,

indicating lower risk. By examining how portfolio variance changes over time, we identify periods of increased or decreased risk. Testing the robustness of the asset as a hedge involves applying the GARCH model across different time periods and market conditions. This step can reveal whether the asset's hedging properties are consistent over time and under varying market scenarios. That's the reason we've selected a relatively long period, including global stress events, as well as events with localized effects in the blockchain domain.

Portfolio variance over time is, on average, close to zero for all observed asset pairs. It is, however, worth mentioning, that the global COVID pandemic has led to a fundamental downturn across the board, and all asset pairs have comparable variance spikes during this time. This leads to the conclusion that while there are many hedging prospects, very few assets remain uncorrelated at times of global systemic risk and uncertainty. Many articles have reviewed the safe haven (both short- and long-term) capabilities of gold or Bitcoin (Baur & McDermott, 2010) (Baur & Lucey, Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold, 2010) their hedging potential is time-varying, regime-dependent and nonlinear, implying that it varies across different regimes. (Adekoya, Oliyide, & Oduyemi, 2021).

Figure 7a: Portfolio variances over time of asset and hedge pairs

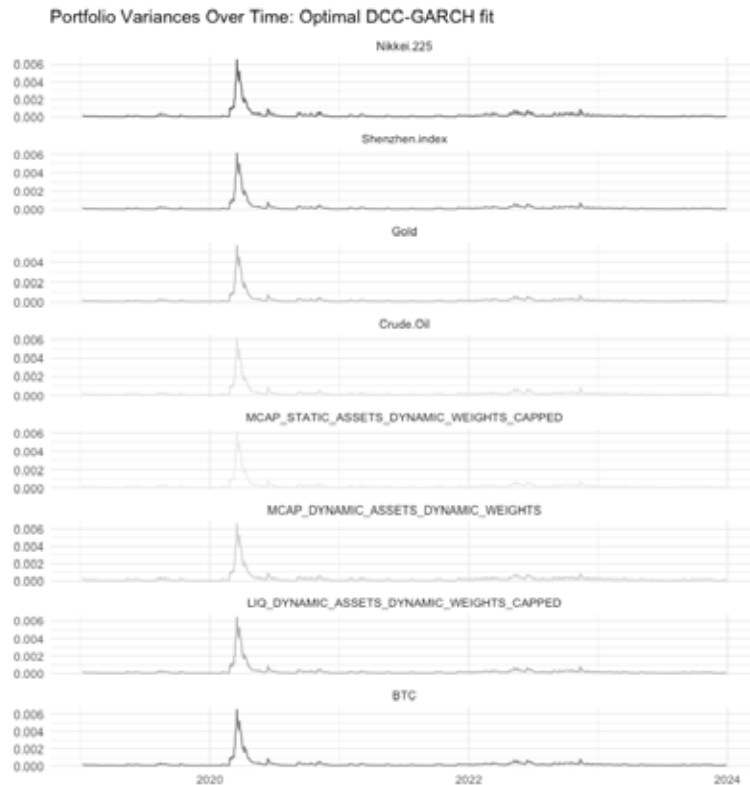
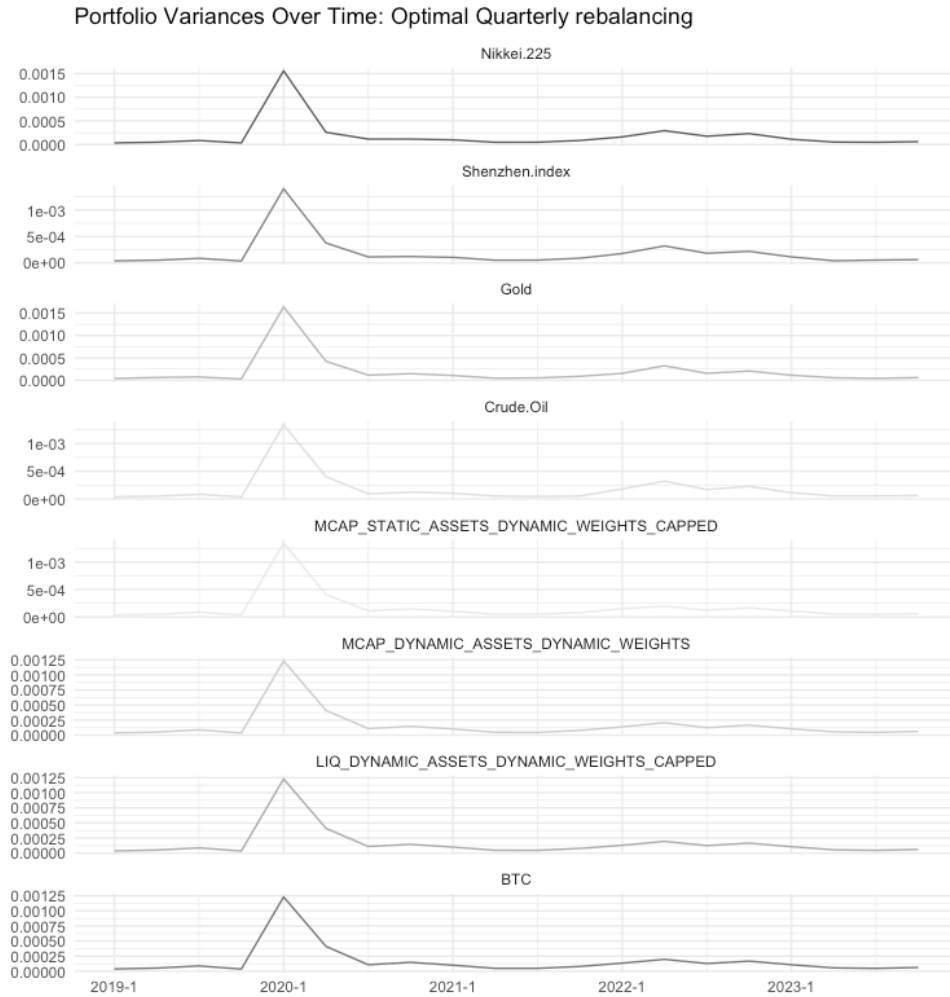


Figure 7b: Portfolio variances by using a quarterly rebalancing strategy



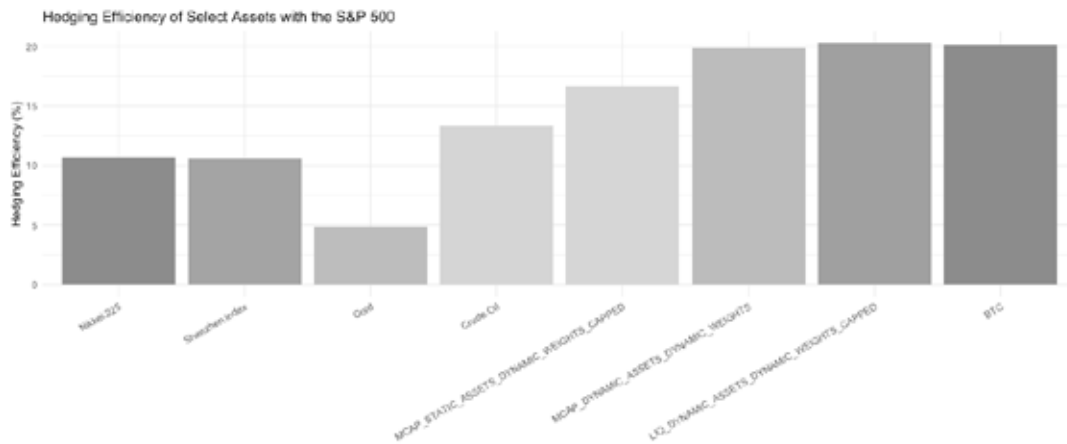
In order to settle the debate, we choose to employ the hedge effectiveness ratio; the performance of optimal hedge ratio estimations is assessed by a hedging effectiveness (HE) index (Arouri et al.⁸, Basher and Sadorsky⁹), through the following formula:

$$HE = \frac{VAR_{Unhedged} - VAR_{Hedged}}{VAR_{Unhedged}} \quad (11)$$

where VAR_{Hedged} denotes the variance of the returns of the S&P 500 portfolio and $VAR_{Unhedged}$ refers to the variance of returns of the blockchain-based portfolio.

A hedging effectiveness of 1 would indicate a perfect hedge that fully eliminates risk; a hedging effectiveness of 0 would indicate a hedge candidate, in fact, provides no risk reduction. Negative hedging efficiency, on the other hand, means that the risks are higher after hedging (the hedge is unsuccessful, as the undesired effects are exacerbated). As we've proven the limited impact of the asymmetric effects, we opt to use DCC GARCH to compare hedge efficiency and returns.

Figure 8: Hedging efficiency of S&P 500 with select assets



As expected, Bitcoin and the overall index demonstrate comparable results. Due to the relatively low computed optimal hedge ratios, Gold appears to have little hedging efficiency for the proposed optimal portfolio; on the other hand, the crypto indices and the core asset – Bitcoin, demonstrate the highest HE across the set of examined assets. Notably, Hedging efficiency is susceptible to overfitting the model.

6. Conclusion

Efficient and affordable hedging of investments in an economy riddled with uncertainty is essential for both institutional and non-institutional investors. Given the accelerated pace of globalization and the expansive policies of central banks, hedging has become more challenging and costly than ever.

This study presents novel approaches to hedging the risks of well-established, previously low-risk investments, such as the S&P 500 index, through the use of indices based on blockchain-based digital assets. Despite an overall positive correlation, which is not ideal, digital asset indices offer hedging efficiencies comparable to – if not slightly better than – other widely utilized instruments. It is vital to acknowledge that digital assets display unique

risk/return characteristics. While risk/return ratios may seem more favourable as observed via the Sharpe ratio, this metric is heavily influenced by the observed period, with short-term losses being a likely occurrence. However, from a mid- or long-term hedging perspective, digital asset indices might prove to be a valuable instrument, although this is also dependent on the risk tolerance of the investor. It is well worth noting that while our proposed index does remediate some of the weaknesses of hedging via Bitcoin directly, it has not proven to be significantly superior. Investors should therefore diligently evaluate if the improvements are worth the operational overhead of managing an index with 12 constituents.

Further research could use the model to forecast a dynamic hedge ratio and further test hedging efficiency, based on forecasted optimal hedging parameter, and compare it with the one calculated based on actuals. Another important aspect is to test the ability and utility of the proposed indices in acting as (weak/short-term) hedges, (weak) safe havens or investment diversifiers. Baur & McDermott have already developed a testing methodology in that respect, which would need to be slightly adapted to fit the cryptocurrency context. (Baur & McDermott, 2010). Lastly, the lack of asymmetricity of the dynamic correlations is rather unexpected. While there might be multiple contributing factors, it is well worth exploring, especially as we see it missing from all hedging pairs.

References

- Adekoya, O. B., Oliyide, J. A., Oduyemi, G. O. (2021) How COVID-19 upturns the hedging potentials of gold against oil and stock markets risks: Nonlinear evidences through threshold regression and markov-regime switching models. – *Resources Policy*, Vol. 70, ISSN 0301-4207.
- Arouri, M. E. H., Jouini, J., Nguyen, D. K. (2021). On the impacts of oil price fluctuations on European equity markets: Volatility spillover and hedging effectiveness. – *Energy Economics*, Vol. 34, N 2, pp. 611-617, ISSN 0140-9883.
- Bakas, D., Magkonis, G., Young Oh, E. (2022). What drives volatility in Bitcoin market? – *Finance Research Letters*, Vol. 50, ISSN 1544-6123.
- Basher, S. A., Sadorsky, P. (2016). Hedging emerging market stock prices with oil, gold, VIX, and bonds: A comparison between DCC, ADCC and GO-GARCH. – *Energy Economics*, Vol. 54, pp. 235-247, ISSN 0140-9883.
- Baur, D. G., McDermott, T. K. (2010). Is gold a safe haven? International evidence. – *Journal of Banking & Finance*, Vol. 34, N 8, pp. 1886-1898, ISSN 0378-4266.
- Baur, D., Lai, H., Hossain, M. Z. (2022). Is Bitcoin a hedge? How extreme volatility can destroy the hedge property. – *Finance Research Letters*, Vol. 47, Part B, ISSN 1544-6123.
- Baur, D., Lucey, B. M. (2010). Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold. – *The Financial Review*, Vol. 45, IN 2, pp. 217-229, ISSN 0732-8516.
- Bauwens, L., Laurent, S., Rombo, J. V. K. (2006). Multivariate GARCH models: a survey. – *Journal of applied econometrics*, Vol. 21, N 1, pp. 79-109, ISSN 0883-7252.
- Bollerslev, T., Engle, R., Wooldridge, J. (1988). A Capital Asset Pricing Model with Time-Varying Covariances. – *Journal of Political Economy*, Vol. 96, N 1, pp. 116-31, ISSN 0022-3808.
- Chan, E. (2021). *Quantitative Trading: How to Build Your Own Algorithmic Trading Business*. 2nd ed. Wiley, ISBN 9-7811-1980-0064.
- Chen, Y.-L., Chang, Y. T., Yang, J. J. (2023). Cryptocurrency hacking incidents and the price dynamics of Bitcoin spot and futures. – *Finance Research Letters*, Volume 55, Part B, ISSN 1544-6123.
- Dong, B., Lei, J., Liu, J., Zhu, Y. (2022). Liquidity in the cryptocurrency market and commonalities across anomalies. – *International Review of Financial Analysis*, Vol. 81, ISSN 1057-5219.
- Dyhrberg, A. H. (2016). Hedging capabilities of bitcoin. Is it the virtual gold?. – *Finance Research Letters*, Vol. 16, pp. 139-144, ISSN 1544-6123.
- Engle, R. (2001). GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics. – *Journal of economic perspectives*, Vol. 15, N 4, pp. 157-168.

- Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. – *Econometrica*, Vol. 50, N 4, pp. 987-1008, ISSN 0012-9682.
- Fakhfekh, M., Jeribi, A. (2020). Volatility dynamics of crypto-currencies' returns: Evidence from asymmetric and long memory GARCH models. – *Research in International Business and Finance*, Vol. 51, ISSN 0275-5319.
- Gerunov, A. (2022). Risk in Digital Assets. – In: Gerunov, A. *Risk Analysis for the Digital Age*. Sofia: Springer Nature Switzerland, ISBN 978-3-031-18100-9.
- Iyer, S. R., Simkins, B. J. (2022). COVID-19 and the Economy: Summary of research and future directions. – *Finance Research Letters*, Vol. 47, Part B, ISSN 1544-6123.
- Jia, Z. et al. (2023). Asymmetric nexus between Bitcoin, gold resources and stock market returns: Novel findings from quantile estimates. – *Resources Policy*, Vol. 81, ISSN 0301-4207.
- Klein, T., Thu, H. P., Walther, T. (2018). Bitcoin is not the New Gold – A comparison of volatility, correlation, and portfolio performance. – *International Review of Financial Analysis*, Vol. 59, pp. 105-116, ISSN 1057-5219.
- Kroner, K., Sultan, J. (1993). Time dynamic varying distributions and dynamic hedging with foreign currency futures. – *Journal of Financial Quantitative Analysis*, Vol. 28, N 4, pp. 535-551, ISSN 0022-1090.
- Lai, Y.-S. (2019). Evaluating the hedging performance of multivariate GARCH models. – *Asia Pacific Management Review*, Vol. 24, N 1, pp. 86-95, ISSN 1029-3132.
- Li, D. et al. (2024). On Stablecoin: Ecosystem, architecture, mechanism and applicability as payment method. – *Computer Standards & Interfaces*, Vol. 87, ISSN 0920-5489.
- Moreno, D., Antoli, M., Quintana, D. (2022). Benefits of investing in cryptocurrencies when liquidity is a factor. – *Research in International Business and Finance*, Vol. 63, ISSN 0275-5319.
- O'Donnell, C. (2022). How to Build Sustainable Productivity Indexes. – School of Economics University of Queensland St. Lucia [online], available at: <https://espace.library.uq.edu.au/view/UQ:863eb20>
- Pal, D., Mitra, S. K. (2019). Hedging bitcoin with other financial assets. – *Finance Research Letters*, Vol. 30, pp. 30-36, ISSN 1544-6123.
- Shah, A., Chauhan, Y., Chaudhury, B. (2021). Principal component analysis based construction and evaluation of cryptocurrency index. – *Expert Systems with Applications*, Vol. 163, ISSN 0957-4174.
- Tiwari, A. K., Raheem, I. D., Kang, S. H. (2019). Time-varying dynamic conditional correlation between stock and cryptocurrency markets using the copula-ADCC-EGARCH model. – *Physica A: Statistical Mechanics and its Applications*, Vol. 535, ISSN 0378-4371.
- Yousfi, M., Farhani, R., Bouzgarrou, H. (2024). From the pandemic to the Russia–Ukraine crisis: Dynamic behavior of connectedness between financial markets and implications for portfolio management. – *Economic Analysis and Policy*, Vol. 81, pp. 1178-1197, ISSN 0313-5926.